Performance Evaluation of a Five-Variable Face-Centered Central Composite Design with Full and Fractional Factorial Points in Process Optimization

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Abstract
Experimental design techniques play a crucial role in optimizing processes, particularly in resource-constrained environments. The Face-Centered Central Composite Design (FCCCD) is widely used for response surface modeling, but its performance when combining full and fractional portions remains underexplored. This study evaluates the performance of a Five-Variable Face-Centered Central Composite Design (FCCCD) with full and fractional factorial points in process optimization. The objective is to compare the design efficiency, predictive accuracy, and reliability of both design types under varying experimental conditions. The study assesses design parameters, fit statistics, and optimality criteria using statistical metrics such as A-efficiency, D-efficiency, and G-efficiency. Model validation is performed to show if the model fits the data, and adequacy precision through residual versus predicted plots. The performance of FCCCD is analyzed in terms of model adequacy, predictive capability, and practical feasibility. The impact of center points on model fit is also investigated. The findings provide that the fractional factorial design demonstrates significant advantages in efficiency, achieving higher A-efficiency (25.20% versus 18.55%) and D-efficiency (32.13% versus 12.06%) compared to the full factorial design. This makes it ideal for studies constrained by time, budget, or experimental resources. Despite its efficiency, it shows mild heteroscedasticity and non-linearity at the extremes of the response range, suggesting potential for further refinement. On the other hand, the full factorial design achieves superior G-efficiency (94.66% versus 80.59%), making it better suited for applications requiring extensive exploration of variable interactions and robust predictions. The fit statistics for FCCCD indicate strong model performance. The full factorial designs with 5 center points shows a moderate coefficient of variation of 38.65%, higher R^2 (94.01%), strong adequacy precision (29.13), though, residual analysis suggests some non-linearity and heteroscedasticity. The fractional factorial designs (1 and 5 center points) exhibit lower coefficient of variation (18.38% and 17.89%), higher R^2 (>98%), and excellent adequacy precision (>36), confirming stability and predictive accuracy. Residual plots reveal slight heteroscedasticity but overall model robustness. These findings offer valuable guidance for selecting appropriate experimental designs in process optimization.

Keywords: *Face-Centered Central Composite Design, Process Optimization, Full Factorial, Fractional Factorial, Optimality Criteria*

Introduction

Process optimization plays a crucial role in industries such as manufacturing, chemical processing, and pharmaceuticals, where enhancing efficiency, improving product quality, and reducing operational costs are key priorities. As industrial processes grow increasingly complex, optimization techniques like Design of Experiments (DoE) and Response Surface Methodology (RSM) have become essential tools for researchers and engineers to analyze variable interactions and determine optimal process conditions. RSM, a widely used statistical approach, helps in constructing mathematical models that describe the relationships between input factors and response variables (Montgomery, 2017). Among the experimental designs in RSM, the Central Composite Design (CCD) is particularly effective, as it enables the development of predictive models that incorporate linear, quadratic, and interaction effects (Box & Draper, 1987). However, despite its effectiveness, CCD often demands a large number of experimental runs, making it both time-intensive and resource-demanding, which can pose challenges in real-world industrial applications. To mitigate the limitations associated with CCD, the Face-Centered Central Composite Design (FCCCD) has been introduced as a more efficient alternative. Unlike traditional CCD, FCCCD strategically positions its axial points on the faces of the experimental design space, thereby reducing the number of experimental runs required while still maintaining model accuracy (Anderson & Whitcomb, 2016). This makes FCCCD particularly beneficial in scenarios where resource efficiency is a primary concern. Additionally, FCCCD is frequently combined with full and fractional factorial designs to achieve a balance between experimental precision and cost-effectiveness. Full factorial designs provide the highest accuracy by evaluating all possible factor combinations but require extensive time and resources. In contrast, fractional factorial designs significantly reduce the number of required runs by analyzing only a subset of factor combinations, although this efficiency gain comes at the risk of aliasing effects, which may obscure important interaction effects (Wu & Hamada, 2009). Despite its efficiency, the robustness of FCCCD in practical industrial settings is influenced by factors such as process variability, equipment inconsistencies, and external noise, which can impact optimization results. While FCCCD has gained popularity for its ability to minimize experimental burden, its effectiveness in handling real-world variability remains an area of active research. Ensuring that FCCCD, particularly in conjunction with full and fractional factorial designs, maintains both predictive accuracy and resource efficiency is crucial for its application in industrial process optimization. Studies indicate that variability and noise in industrial operations can significantly influence optimization outcomes, highlighting the need to assess FCCCD's reliability and reproducibility under such conditions (Montgomery, 2017). In modern industries, there is an increasing demand for efficient optimization methods that improve productivity, enhance product quality, and lower operational expenses. While traditional designs like CCD offer valuable insights, their extensive experimental requirements can make them impractical in resource-constrained environments. FCCCD provides a more resource-efficient alternative, yet its integration with fractional factorial designs introduces challenges such as potential aliasing and information loss regarding critical effects. Furthermore, the impact of uncontrolled variability in industrial settings has not been extensively explored, leaving a gap in understanding FCCCD's real-world applicability (Wu & Hamada, 2009).

Therefore, this study focused on assessing the efficiency and predictive accuracy of FCCCD when integrating full and fractional factorial designs in optimizing complex industrial processes. This research contributes to the advancement of modern process optimization methodologies that support industrial growth and economic efficiency. Effective optimization techniques help industries enhance competitiveness, minimize material waste, and create employment opportunities, ultimately promoting sustainable industrial development (Anderson & Whitcomb, 2016). As industries continue to seek innovative and cost-effective solutions, the findings of this study will provide valuable insights into the potential of FCCCD as a practical and efficient approach to process optimization.

Several studies have emphasized the advantages of integrating fractional factorial designs within response surface methodology (RSM). Alvarez et al. (2009) demonstrated that fractional factorial designs enhance experimental efficiency by reducing the number of runs while maintaining model accuracy. Their study reported improvements in A-efficiency and D-efficiency, indicating that fractional factorial designs optimize resources without compromising predictive performance. Similarly, Singh et al. (2017) compared Full Factorial and Fractional Factorial Designs, showing that fractional designs effectively manage multiple variables while maintaining accuracy, making them suitable for complex experimental setups. Kumar et al. (2018) used FCCCD in a five-variable machining process to improve surface roughness, demonstrating its capability to enhance process performance systematically. Patel et al. (2020) conducted a comparative study between FCCCD and traditional CCD, evaluating prediction accuracy and mean square error (MSE). Their findings confirmed that FCCCD outperforms CCD in optimization efficiency and is more robust against data variability, making it a preferred choice for industrial applications. Li et al. (2019) examined the impact of outliers on CCD and proposed robust regression techniques, such as the least absolute deviation (LAD) estimator, to mitigate distortions in model predictions. Li et al. (2020) extended this research by comparing FCCCD with BBD and PBD, demonstrating that FCCCD exhibits superior optimization efficiency and noise resistance, even in high-variability environments. These findings reinforce FCCCD's applicability in scenarios where unpredictable factors influence process outcomes. Box and Wilson (1951) introduced CCD as a systematic approach for optimizing processes using quadratic response surfaces. Khuri and Cornell (1987) later evaluated FCCCD's robustness in managing non-normality and unequal variance, demonstrating its adaptability across different data distributions. More recently, Iwundu and Cosmos (2022) explored the trade-offs between model complexity and efficiency in a seven-variable BBD, emphasizing the importance of selecting an optimal number of center points for improved model performance. Wu and Hamada (2000) compared FCCCD with other experimental designs, concluding that it remains a reliable and flexible approach for process optimization. Montgomery (2017) further highlighted FCCCD's capability in addressing non-linear relationships while reducing errors and outliers. Similarly, Anderson and Whitcomb (2016) confirmed FCCCD's robustness in managing variable interactions, making it ideal for experimental designs involving multiple influencing factors. Despite the wealth of research on FCCCD, an important gap remains regarding the integration of full and fractional factorial portions within FCCCD using different numbers of center points. While prior studies have explored FCCCD's effectiveness, robustness, and efficiency, none have explicitly examined how the combination of full and fractional factorial portions with 1 and 5 center points affects experimental outcomes. This gap presents an opportunity for further research to explore FCCCD's potential in optimizing experimental designs while maintaining accuracy and resource efficiency.

Materials and Methods

Design

This study utilized a Five-Variable Face-Centered Central Composite Design (FCCCD) to explore and optimize process conditions. The design incorporated:

Full Factorial Points: Representing all combinations of factor levels (2^K), where k is the number of factors. For five variables, this resulted in 2^5 = 32 experimental runs at high (+ 1) and low (- 1) levels.

Fractional Factorial Points: A half-fractional factorial design (2^{K-1}) was integrated to reduce the number of runs to 16 while capturing significant main effects and key interactions.

Center Points: Center points (midpoint of factor ranges, coded as 0) were added to detect curvature and assess model robustness. Experiments with 1 and 5 center points were included for comparison.

The FCCCD's structure allowed exploration of linear, quadratic, and interaction effects while maintaining resource efficiency.

Statistical Methods

Key statistical methods and their respective formulas used for data analysis are detailed below:

Optimality Criteria

A-optimality: Measures the average variance of the predicted responses, minimizing parameter estimate variances.

$$A = \frac{Trace(X^T X)^{-1})}{K}$$

where X = design matrix,

k = number of factors

D-optimality: Maximizes the determinant of the information matrix $(X^T X)$, ensuring minimal parameter estimation variance.

$$D = (\det(X^T X))^{1/k}$$

G-optimality: Minimizes the maximum prediction variance across the design space.

G = max(h(x), where h(x) = $x^T(X^TX)^{-1}x$

Model Validation Metrics

R-squared (R^2): Measures the proportion of variance in the response variable explained by the model.

$R^2 = 1 - \frac{SS_R}{SS_T}$

where:

SS_R is the sum of squares residuals (also known as sum of squared error). It measures the unexplained variability in the model.

SS_T is the total sum of squares, which represent the total variation in the observed data.

However, R^2 ranges between 0 and 1. Then, $R^2 = 1$, indicates a perfect fit, where the model explains all the variability in the response variable. R^2 = 0, indicates that the model does not explain any of the variability; the predictions are no better than the mean of the observed data. Also, 0 < R^2 < 1, indicates the proportion of the variability in the dependent variable that is explained by the independent variables. Thus, the higher the R^2 value, the better the model fits the data.

Predicted R-squared ($R^2_{pred.}$): Evaluates the model's predictive ability on new data.

PRED. R^2 ($R^2_{pred.}$) = 1 - $\frac{PRESS}{SS_{Total}}$

where: PRESS is Predicted Residual Error Sum of Squares = $\sum_{i-1}^{n}(y_i - \hat{y}_{\{i\}}).^2$

SS_{Total} is the sum of squares total.

However, a higher $R^2_{pred.}$ close to 1 suggests that the model has a strong predictive accuracy, while a low $R^2_{pred.}$ indicates that the model may not generalize well to new data.

Adequacy Prediction: The adequacy precision evaluates the signal-to-noise ratio of a model. It helps to determine if the model's predictions are reliable.

Adequacy precision is defined as:

Adequacy Precision = $\frac{Range\ of\ predicted}{Average\ prediction\ error}$ = $\frac{\hat{y}_{max} - \hat{y}_{min}}{\sqrt{\frac{1}{n}\sum_{i=1}^{n}(yi - \hat{y})^2}} > 4$

where: Range of predicted values: Is the difference between the maximum and minimum predicted values, Average predicted error: Typically represented by the Root Mean Square Error (RMSE)

y_i = actual observed values, ŷ = predicted values from the model, n = number of data points (observations), Adequacy precision > 4: Suggests that the signal is much larger than the noise, meaning the model's predictions are reliable.

Adequacy precision < 4: Suggests that the noise (random error) in the data may be overwhelming the signal, implying that the model may not be very reliable.

Alternatively; Adequacy Precision = $\frac{Signal}{Noise}$

where; Signal represents the range of predicted responses over the design space, Noise represents the noise level, which is the average error or deviation from the predicted response.

Residual Analysis: Residuals (e_i) were analyzed to check normality and homoscedasticity.

$e_i = y_i - \widehat{y_i}$

Graphs like Normal Plot of Residuals, Residuals vs. Predicted Plot, and Residuals vs. Run Plot were used to validate assumptions.

Justification

The selected FCCCD with full and fractional factorial points provided a robust framework to investigate linear, quadratic, and interaction effects. A-optimality, D-optimality, and G-optimality ensured precision, efficiency, and predictability, aligning with the study's goal of optimizing complex processes. Model validation metrics and residual analysis guaranteed reliability and generalizability of results, ensuring minimal experimental runs without compromising accuracy.

Models

Full Quadratic Model

The full quadratic model captures the linear, interaction, and quadratic effects of the factors. It is given as:

$$Y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{i<j} \sum \beta_{ij} x_i x_j + \varepsilon_{ij}$$

where: Y: Dependent (response) variable, β_0 : Intercept term, β_i : Coefficient term, β_{ij} : Coefficients of interaction terms (where $i \ddagger j$), β_{ii} : Coefficient of quadratic terms, x_i and x_j are the independent variables, ε_{ij}: Error terms

Reduced Quadratic Model

To improve efficiency and reduce complexity, insignificant terms are removed after statistical analysis. The reduced quadratic model includes only significant linear, interaction, and quadratic terms:

$$Y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{ij} \beta_{ij} X_i X_j + \varepsilon$$

First-Order (Linear) Model

The first-order model is used when there is no significant curvature in the response surface. It is given by:

Y = β_0 + $\sum_{i=1}^{k} \beta_i X_i + \epsilon$

Second-Order (Pure Quadratic) Model

The second-order model focuses on quadratic effects without interaction terms:

Y = β_0 + $\sum_{i=1}^{k} \beta_{ii} X_i^2$ + ϵ

Interaction Model

This model focuses solely on the interaction terms between variables:

Y = β_0 + $\sum_{i=1}^{k} \sum_{j=i+1}^{k} \beta_{ij} X_i X_j$ + ϵ

These models collectively allow for flexible analysis of linear, quadratic, and interaction effects in the FCCCD framework. Depending on the significance of factors and their combinations, the appropriate model is chosen for analysis and optimization.

The fitted polynomial model, considering the five-variables X1, X2, X3, X4, X5 are as follow:

Linear Terms:

$$\beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5$$

Interaction Terms:

$$\beta_{12}X_1X_2 + \beta_{13}X_1X_3 + \beta_{14}X_1X_4 + \beta_{15}X_1X_5 + \beta_{23}X_2X_3 + \beta_{24}X_2X_4 + \beta_{25}X_2X_5$$

Quadratic Terms:

$$\beta_{11}X_1^2 + \beta_{22}X_2^2 + \beta_{33}X_3^2 + \beta_{44}X_4^2 + \beta_{55}X_5^2$$

Bring equation 3.3, 3.4, and 3.5 together, they form the basis for fitting the model to the data and making predictions. Thus, the fitted quadratic (second-order) polynomial model can be written as:

$$y = \beta_1X_1 + \beta_2X_2 + \beta_4X_4 + \beta_5X_5 + \beta_{12}X_1X_2 + \beta_{13}X_1X_3 + \beta_{14}X_1X_4 + \beta_{15}X_1X_5 + \beta_{23}X_2X_3 + \beta_{24}X_2X_4 + \beta_{25}X_2X_5 + \beta_{11}X_1^2 + \beta_{22}X_2^2 + \beta_{33}X_3^2 + \beta_{44}X_4^2 + \beta_{55}X_5^2$$

This combination forms the regression model, where the design matrix X provides the structure for fitting the data, and the model equation captures the relationships among the factors and the response variable. Therefore, the model matrix associated with central composite design in k design variables for axial distance α and design size, N is represented in Algebraic form:

X =

$$\begin{bmatrix}
1 & x_{11} & x_{12} \cdots & x_{1k} & x_{11}^2 & x_{12}^2 \ldots & x_{1k}^2 & x_{11}x_{12} & x_{11}x_{13} & \cdots & x_{1(k-1)}x_{1k} \\
1 & x_{21} & x_{22} \cdots & x_{2k} & x_{21}^2 & x_{22}^2 \ldots & x_{2k}^2 & x_{21}x_{22} & x_{21}x_{23} & \cdots & x_{2(k-1)}x_{2k} \\
1 & x_{31} & x_{32} \cdots & x_{3k} & x_{31}^2 & x_{32}^2 \ldots & x_{3k}^2 & x_{31}x_{32} & x_{31}x_{33} & \cdots & x_{3(k-1)}x_{3k} \\
1 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots \\
1 & x_{n1} & x_{n2} \cdots & x_{nk} & x_{n1}^2 & x_{n2}^2 \ldots & x_{nk}^2 & x_{n1}x_{n2} & x_{n1}x_{n3} & \cdots & x_{n(k-1)}x_{nk} \\
1 & \alpha & 0 & 0 & \alpha^2 & 0 & 0 & 0 & 0 & & 0 \\
1 & \alpha & 0 & 0 & \alpha^2 & 0 & 0 & 0 & 0 & \cdots & 0 \\
1 & 0 & \alpha & 0 & 0 & \alpha^2 & 0 & 0 & 0 & & 0 \\
1 & 0 & \alpha & 0 & 0 & \alpha^2 & 0 & 0 & 0 & \cdots & 0 \\
1 & 0 & 0 & \alpha & 0 & 0 & \alpha^2 & 0 & 0 & & 0 \\
1 & \vdots & \vdots & \alpha & \vdots & \vdots & \alpha^2 & \vdots & \vdots & & \vdots \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0
\end{bmatrix}$$

……. (3.7)

where: X is the design matrix, n is the number of experimental runs, K is the number of factors (independent variables), X_{ij} represents the level of factor ^{j} in run ^{i}.

Hence, the design metrics are to optimize a response, efficient exploration of factors and their interactions, fitting a second-order polynomial model for prediction, reducing number of experimental runs needed, predicting system behavior based on the model, identifying optimal conditions for the process, and assessing robustness factor settings.

Results and Discussion

The analysis results are presented in Tables 1 to 5 and Figures 1 to 4. For better clarity, the design efficiency values, fit statistics, and diagnostic properties of the full and fractional quadratic models are summarized under different center points to facilitate easy comparison.

Table 1: Design Efficiency Values for FCCCD with 2^5 and 2_V^{5-1} Factorial Designs

FCCCD	A-Efficiency	D—Efficiency	G-Efficiency
2^5	18.55%	12.06%	94.66%
2_V^{5-1}	25.20%	32.13%	80.59%

Table 2: Fit Statistics for FCCCD with 2^5 Factorial Design and 1 Center Point

Std.Dev.	2042.01
Mean	5166,68
C.V. %	39.2
R^2	0.9401
AdjustedR^2	0.9189
PredictedR^2	0.8753
Adequacy Precision	26.33

Table 3: Fit Statistics for FCCCD with 2^5 Factorial Design and 5 Center Points

Std.Dev.	1929.32
Mean	4991.33
C.V.%	38.65
R^2	0.9401
Adjusted R^2	0.9213
Predicted R^2	0.8762
Adequacy Precision	29.13

Table 4: Fit Statistics for FCCCD with 2_V^{5-1} Factorial Design and 1 Center Point

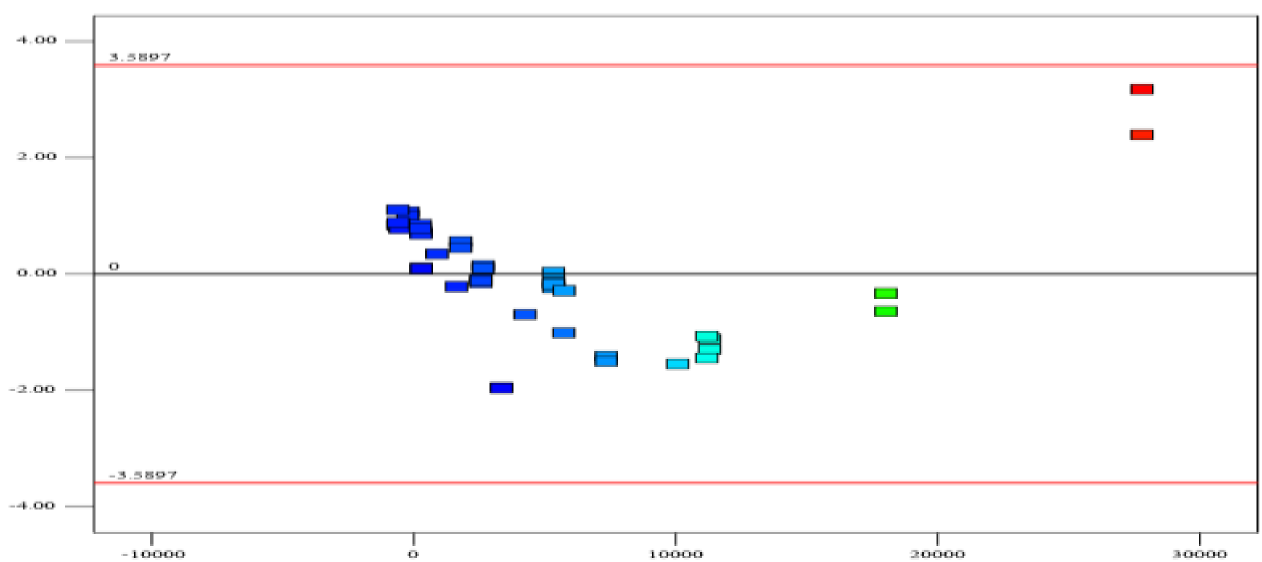
Std.Dev.	665.37
Mean	3619.45
C.V.%	18.38
R^2	0.9828

AdjustedR^2	0.9694
PredictedR^2	0.9066
Adequacy Precision	36.9140

Table 5: Fit Statistics for FCCCD with 2_V^{5-1} Factorial Design and 5 Center Points

Std.Dev.	663.03
Mean	3706.59
C.V.%	17.89
R^2	0.9807
AdjustedR^2	0.9650
PredictedR^2	0.9588
Adequacy Precision	37.21

Resiuals



Predicted

Figure 1: Residual Versus Predicted for FCCCD with 2^5 Factorial Design and 1 Center Point.

Residual

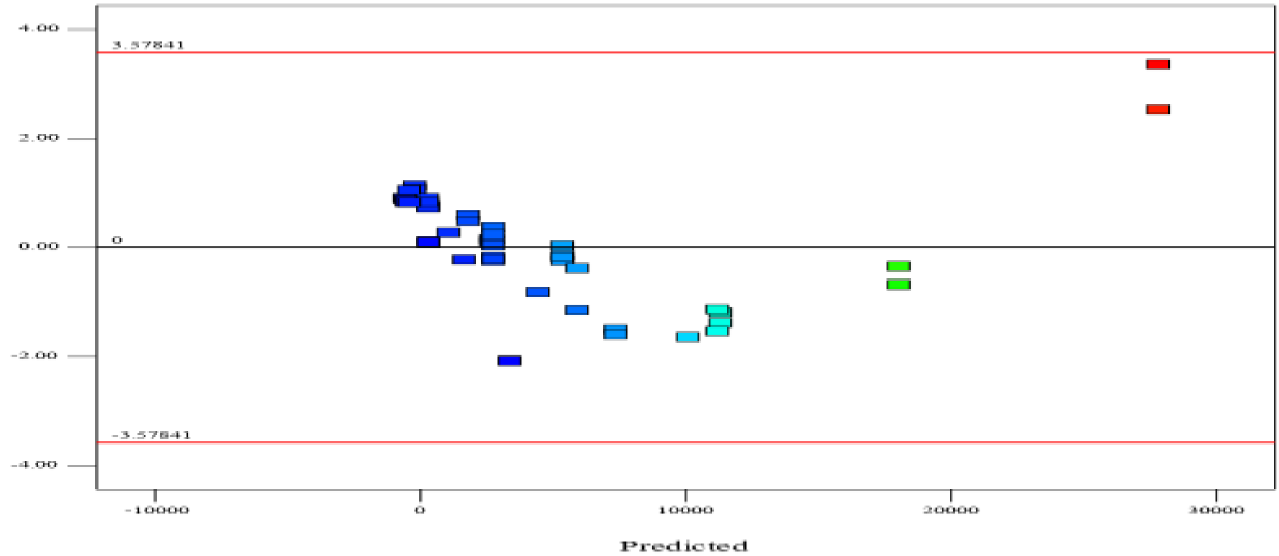
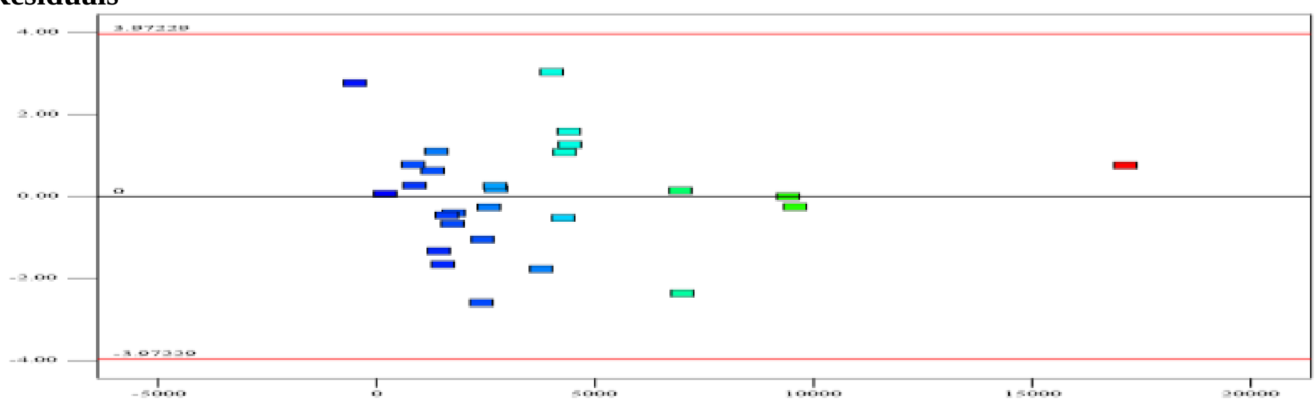


Figure 2: Residual Vs Predicted for FCCCD with 2^5 Factorial Design and 5 Center Points.

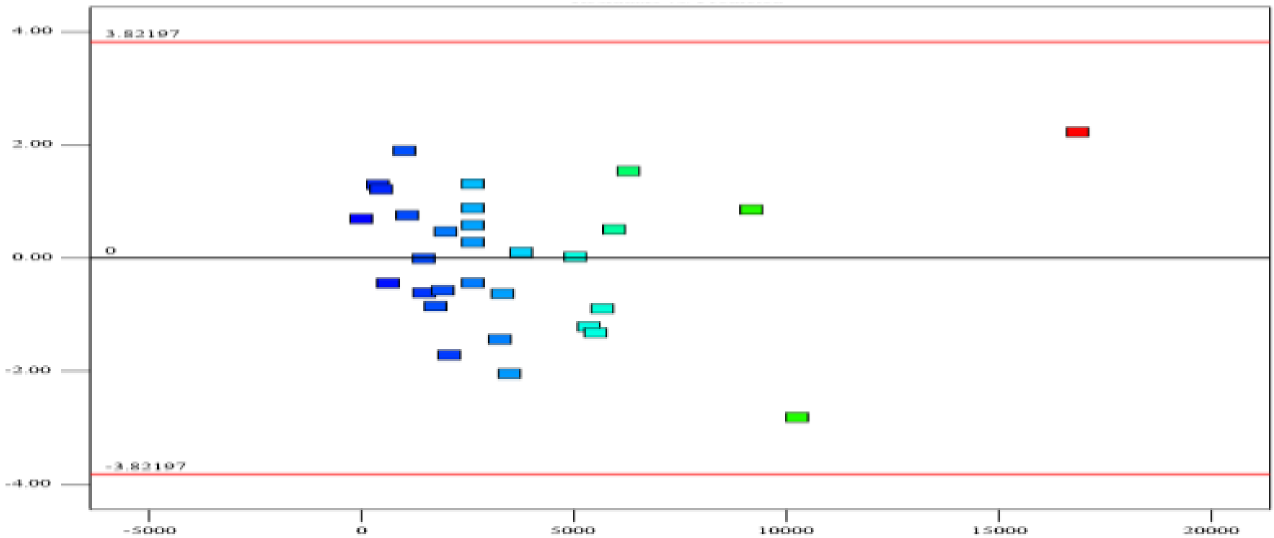
Residuals



Predicted

Figure 3: Residual Vs Predicted for FCCCD with 2_V^{5-1} Factorial Design and 1 Center Point.

Residual



Predicted

Figure 4: Residual Vs Predicted for FCCCD with 2_V^{5-1} Factorial Design and 5 Center Points.

Discussion of Findings

The discussion delves into key findings from the design efficiency values, fit statistics, and residual analyses, highlighting the performance of both full and fractional factorial FCCCDs under varying conditions. The results provide valuable insights into the strengths and limitations of these designs, which are essential for process optimization and experimental planning.

Design Efficiency Values

The analysis of A-, D-, and G-efficiency values underscores the trade-offs between full and fractional factorial designs:

1. A-Efficiency

Fractional factorial designs exhibit higher A-efficiency (25.20%) compared to full factorial designs (18.55%), suggesting better resource utilization in estimating model coefficients. This is particularly advantageous in resource-constrained settings where fewer experimental runs are desired without compromising estimation precision. The fractional design's ability to isolate key variables efficiently positions it as a practical choice for exploratory studies or situations requiring economical experimentation.

2. D-Efficiency

The significantly higher D-efficiency of fractional designs (32.13%) compared to full factorial designs (12.06%) indicates superior optimization for precise coefficient estimation. This highlights the practicality of fractional designs in applications where minimizing the number of runs is critical, such as time-sensitive or cost-intensive experiments. Fractional designs effectively balance experimental economy with accuracy in coefficient determination.

3. G-Efficiency

Full factorial designs outperform fractional designs in G-efficiency (94.66% vs. 80.59%), indicating better coverage of the design space and lower prediction variance. This robustness is vital for applications requiring comprehensive exploration and reliable predictions across a wide range of parameter values. However, the slightly lower G-efficiency of fractional designs is acceptable for targeted investigations focusing on key variables and interactions.

Fit Statistics

The fit statistics reveal strong model performance across both designs, but some areas warrant attention:

1. High Predictive Power

Both full and fractional designs exhibit high R^2 values (>94%), adjusted R^2 values (>91%), and predicted R^2 values (>87%). These metrics confirm the models' strong ability to capture variability and make accurate predictions. Fractional designs also demonstrate slightly lower coefficient of variation (C.V.), suggesting more consistent predictions with fewer experimental runs.

2. Precision and Reliability

The adequacy precision values exceed the threshold of 4 for all designs, confirming reliable signal-to-noise ratios. Fractional designs achieve higher precision, making them well-suited for efficient exploration and optimization in constrained scenarios.

3. Moderate Variability

The standard deviation and C.V. values for both designs indicate moderate variability. While acceptable, there is scope for refinement to reduce residual dispersion further, enhancing the models' predictive accuracy and consistency.

Residual Analysis

The residuals versus predicted plots provide insights into the models' adherence to assumptions:

1. Heteroscedasticity in Full Factorial Designs

Residual patterns and funnel shapes suggest heteroscedasticity and non-linearity, indicating that the model does not fully capture relationships across the design space. These issues could lead to biased predictions and necessitate modifications such as transformations or additional interaction terms.

2. Robustness in Fractional Factorial Designs

Fractional designs show more random scatter of residuals around the zero line, meeting assumptions of linearity and homoscedasticity. This ensures reliable and unbiased predictions, particularly for targeted or resource-limited applications. However, mild heteroscedasticity in some regions suggests potential improvement through variance stabilization techniques.

Strengths

Fractional designs exhibit superior A- and D-efficiency, indicating better resource economy and precise coefficient estimation with fewer runs. High R^2 and adequacy precision values across all designs confirm strong explanatory and predictive power.

Fractional designs provide consistent predictions (lower C.V.), making them ideal for applications with limited resources.

Limitations

Full factorial designs exhibit non-linearity and heteroscedasticity, necessitating model refinement to address prediction bias.

Fractional designs’ slightly lower G-efficiency reflects limited design space coverage, which may affect reliability for exhaustive exploration.

Conclusion

The findings emphasize the trade-offs between full and fractional factorial FCCCDs. Fractional designs offer practical advantages in resource-limited or exploratory scenarios, excelling in efficiency and predictive accuracy. Full factorial designs, with their superior G-efficiency, are more suitable for comprehensive studies requiring robust predictions across a wide range of parameters. The insights provided guide the selection and refinement of experimental designs based on specific research goals and constraints.

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